PRIORITIES *in* PRACTICE

The Essentials of Mathematics K–6

Effective Curriculum, Instruction, and Assessment

Kathy Checkley

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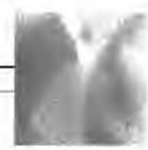

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Gail Underwood, Grant Elementary School, Columbia, Missouri

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Introduction

Math is like love—a simple idea, but it can get complicated.

—*Anonymous*

Is it important that students learn mathematics?

Anyone answer no? It's unlikely. Indeed, mathematics has always had a guaranteed spot in the K–12 curriculum, unlike some of its step-sibling content areas, such as physical education, music, and art.

Pose a slightly different question, however, and the answer may not be so quick or definitive:

Is learning mathematics a cultural imperative?

Before you answer yes, consider what happens when mathematics knowledge—or lack thereof—is put to the test:

You're at a party and, with an apologetic laugh, you admit that the ability to do math is beyond you—in fact, you have a hard time even balancing your checkbook! Among the crowd, one or two people may nod in sympathetic understanding.

Now, imagine that, at that same party, you instead confess that you can't read or write.

The reaction, says Thomas Armstrong, would be quite different. He uses this scenario to underscore the importance placed on literacy in the United States. "Not to be able to read in our culture is a source of shame and humiliation for many," he writes in *Multiple Intelligences of Reading and Writing: Making the Words Come Alive* (2003, p. 5).

Does mathematical illiteracy produce the same disquiet? It's often *said* that a sound mathematics education is key to success in life, but a prevailing ambivalence about whether all students need to become skilled in the subject may suggest otherwise.

"For no other subject in the school curriculum is an adult so proud of having done poorly; failure in mathematics is almost a badge of honor," writes Alfred S. Posamentier in "Marvelous Math!" (2004, p. 44). Posamentier, the dean of the school of education at City College of New York, notes that too many adults "seldom remember math class" as a place where they experienced "learning highs."

What Is Mathematical Literacy?

To be literate in mathematics means that one possesses procedural and computational skills, as well as a conceptual understanding of mathematical concepts.

Mathematical literacy today means mastering the "traditional" basics—achieving computational fluency—and attaining a basic understanding of algebra, geometry, measurement, and data analysis and probability.

Source: Adapted from *Administrator's Guide: How to Support and Improve Mathematics Education in Your School* (pp. 3, 28), by A. J. Mirra, 2003, Reston, VA: National Council of Teachers of Mathematics.

Do parents want their children to learn math? When asked, most parents will always answer yes, says Barbara J. Reys, distinguished professor of mathematics education at the University of Missouri. She adds, however, that some parents will say, "I wasn't good at math"; some parents also believe that not all students can learn mathematics and that "if they don't, it's okay," says Reys.

This vacillation may stem from parents' own unfavorable experiences with math in school, states Reys. It can also result from another kind of angst: Unlike with reading, "it's not too long before kids are studying math that's beyond what their parents studied. Parents are [then] out of the picture," she observes. Educators, too, have communicated mixed signals about how necessary it is for all students to learn mathematics, especially upper-level subjects like algebra and calculus. "Until recently, many people thought about mathematics as a discipline that is comprehensible to only a select, talented few," write Lynn T. Goldsmith and Ilene Kantrov in *Guiding Curriculum Decisions for Middle-Grades Mathematics*. "Instructional traditions paid little attention to helping students make sense of the mathematical ideas they encountered," the authors state (2001, p. 37).

Researcher John H. Holloway agrees, adding that there has also been an instructional tradition of holding minority students to lower expectations. "Minority students as a group experience a less rigorous curriculum," which leads to fewer opportunities to succeed in "gateway courses," such as Algebra 1, he writes in "Research Link: Closing the Achievement Gap in Math" (2004, p. 84).

Slow, But Steady, Change

The societal acceptance of poor mathematics achievement is waning. Dissatisfaction with poor math performance has "become intense and it is growing," write the authors of *Mathematical Proficiency for All Students*. "Every student now needs competency in mathematics," the RAND Mathematics Study Panel asserts. To become mathematically competent, students need to acquire specific knowledge, skills, abilities, and beliefs.

According to researchers at the National Research Council, there are five aspects of mathematical knowledge—five strands—that children must develop throughout their elementary school and middle school years. The five strands, described in *Helping Children Learn Mathematics* (2002, National Academy Press), are interwoven and interdependent. They are

• **Understanding:** This involves comprehending mathematical concepts, operations, and relations, as well as knowing what mathematical symbols, diagrams, and procedures mean. When students have a sound grasp of fundamental mathematical ideas, they know more than isolated facts and procedures. Students know why a mathematical idea is important and when a particular concept is useful. When students truly understand, they are aware of the many connections between mathematical ideas.

• **Computing:** This includes being able to accurately use, with ease, procedures for adding, subtracting, multiplying, and dividing mentally or with paper and pencil. It also involves knowing when and how to use these procedures appropriately. Additionally, the National Research Council broadened its definition of computing from its original meaning—the ability to do arithmetic—to its new definition—the ability to use other mathematical procedures, such as measuring lengths, solving algebraic equations, constructing similar geometric figures, and interpreting and graphing data, like statistics.

• **Applying:** A concept or procedure is not useful unless students recognize when and where to use it—as well as when and where it does not apply. Outside school, students encounter real-life situations and must determine what kind of problem is inherent in each situation. Students, therefore, must be able to discern problems, devise solution strategies, and choose the most useful of those strategies. What's more, students must develop an ability to estimate quantities in their minds or draw them on paper, and they need to know how to distinguish what is known and relevant from what is unknown and irrelevant.

- **Reasoning:** Reasoning is the glue that holds mathematics together. By thinking about the logical relationships between concepts and situations, students can navigate through the elements of a problem and see how they fit together.

- **Engaging:** Engaging in mathematical activity is the key to success. Students should embrace the idea that mathematics makes sense and that—given reasonable effort—they can learn it and use it, both in school and outside school. (Kilpatrick & Swafford, 2002)

Mathematically proficient students know that math is sensible, useful, and worthwhile—and they know that their efforts to learn it will pay off. When students are engaged in mathematics, they know they are effective learners, doers, and users of mathematics. "This goal of achieving mathematical proficiency for all students is unprecedented" (RAND, 2003, p. 2).

A new consciousness about the importance of math has started to "bubble up," notes Cathy Seeley, president of the National Council of Teachers of Mathematics (NCTM). She credits educators' knowledge of the achievement gap, the rise in technology, and "an inundation of information that comes in a statistical and quantitative form," as some of the main factors in helping to shift attitudes.

Jim Bohan credits the standards movement, which NCTM helped to spur, for the increased focus on improving achievement in math (*see A Standards Primer, p. 8*). Bohan is the K–12 mathematics program coordinator in the Mainheim Township School District in Lancaster, Penn. "There are tests now," and there's

"Standards . . . have helped validate the importance of math."
—Jim Bohan, A Standards Primer

A Standards Primer

When the National Council of Teachers of Mathematics (NCTM) published a series of standards documents in 1989, the council signaled "shifts in emphasis" in math education in curriculum, instruction, and assessment, writes Jim Bohan in *Mathematics: A Chapter of the Curriculum Handbook* (2002):

> • Curriculum would emphasize a deeper study of mathematical ideas and concepts and how they are applied today.
>
> • Learning would become more active, students would become more involved with mathematics, and all students would be given opportunities to reach their mathematical potential.
>
> • Student achievement would be assessed through many sources of evidence. (p. 1)

Many educators and professionals accepted the standards, but "resistance to both the vision and implementation of the NCTM standards became substantial and well-organized," Bohan writes (2002, p. 1).

In response to this opposition—as well as to "changes in society, and to the greater understanding of children and learning" (Bohan, 2002, p. 1)—NCTM published the *Principles and Standards for School Mathematics*. Released in 2000, the new standards are intended to be a resource to mathematics curriculum planners, Bohan explains.

According to Bohan, the NCTM standards can help educators develop a mathematics education program that helps students acquire skills and processes that go beyond "computational or symbolic manipulating prowess." That kind of program meets business and industrial expectations of school mathematics. As early as 1987, Bohan writes, Bell Laboratories mathematician Henry Pollack indicated that students should graduate from school with

- The ability to set up problems with appropriate operations.
- A knowledge of a variety of techniques to approach and work on problems.
- An understanding of the underlying mathematical features of a problem.
- The ability to work with others on problems.
- The ability to see the applicability of mathematical ideas to common and complex problems.
- A skill for working with messy problem situations because most real problems are not well formulated.
- The belief in the utility and value of mathematics. (Bohan, 2002, p. 3)

A standards-based mathematics program can help students attain these skills, Bohan maintains. "In many places in the United States, the move to a standards-based math program has produced students who are indeed empowered with the mathematical knowledge to meet any challenge in the future" (2002, p. 4).

Source: Adapted from Mathematics: *A Chapter of the Curriculum Handbook* (pp.1–4), by J. Bohan, 2002, Alexandria, VA: Association for Supervision and Curriculum Development.

accountability. The importance of math, Bohan says, has been validated. And none too soon, many educators assert.

The results of international studies released in December 2004 suggest that mathematics education reform in the United States has been successful on some fronts, but disappointing on others. One such success: 8th grade students who participated in the Trends in International Mathematics and Sciences Study (TIMSS) scored better in both math and science then they had previously. One disappointment: the scores of 15-year-olds who took the Program for International Student Assessment (PISA) were below average in mathematics literacy and problem solving (Association for Supervision and Curriculum Development, 2004/2005).

Perhaps even more important than performance on international tests is the fact that education and career opportunities, as well monetary success, are directly linked to mathematics achievement. The research shows that

- Students who completed higher-level mathematics courses in high school were more likely to earn a bachelor's degree. A longitudinal study conducted by Clifford Adelman, a senior research analyst for the U.S. Department of Education, found that eight percent of high school graduates with algebra 1 under their belts earned a bachelor's degree by age 30. In contrast, 80 percent of those who completed calculus in high school earned a bachelor's degree by age 30 (Adelman, 1999).

- More than half the workers earning more than $40,000 a year had completed two or more credits at the Algebra 2 level or higher. This is according to Anthony P. Carnevale and Donna M. Desrochers of the Educational Testing Service, who analyzed data from the National Educational Longitudinal Survey (Carnevale & Desrochers, 2002).

- Taking higher-level math courses can boost a young person's earning potential after high school, Heather Rose and Julian R. Betts report in *Math Matters: The Links Between High School Curriculum, College Graduation, and Earnings*. Rose and Betts, of the Public Policy Institute of California, found that after controlling for students' demographic, family, and high school characteristics, one extra course in algebra or geometry is associated with 6.3 percent higher earnings (Rose & Betts, 2001).

"Math opens up career paths, empowers consumers, makes meaningful all kinds of data, from basketball statistics to political polls to the latest trends in the stock market," write Harvey F. Silver and Richard W. Strong in the forward to *Styles and Strategies for Teaching Middle School Mathematics* (Silver & Strong, 2003, p. 5). For all these positive outcomes, however, Silver and Strong note a troubling reality: the longer a majority of students are in school, the less they trust in their ability to do math. The authors point out that more than three quarters of all students who graduate from high school don't believe that they are among the "special realm" of people who can be successful in a field that requires in-depth mathematics knowledge.

And that's a serious problem. "If we send an army of math-haters out into today's competitive global culture, we are short-changing millions of students by severely limiting their chances of future success," warn Silver and Strong (2003, p. 5). One response, therefore, would be to create an army of math-lovers—among students, teachers, administrators, and parents. The question is how.

That's the reason for this book—to showcase effective curriculum, instruction, and assessment in mathematics. In doing so, we will

- Give teachers ideas for instructional and assessment approaches that they will want to enthusiastically adopt.
- Share examples of curricula that can inspire a love of mathematics among children and young adults.
- Give administrators ideas for creating a school climate that supports high mathematics achievement for all students.

We will also share examples of professional development that is effective in helping build and enhance our teachers' knowledge of the mathematics they teach.

"The importance of mathematical literacy and the need to understand and use mathematics in everyday life and in the workplace has never been greater and will continue to increase," writes Amy J. Mirra in *Administrator's Guide: How to Support and Improve Mathematics Education in Your School* (2003, p. 1). We hope this book will help educators address the challenge of providing a sound mathematics education for all students. It's nothing short of imperative.